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FINITE DEFORMATION EFFECTS IN HOMOGENEOUS AND INTERFACIAL FRACTURE

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Abstract—This paper summarizes recent results of asymptotic, numerical and experimental investigations of some nonlinear effects on the mechanics of fracture in homogeneous and bimaterial sheets of a particular class of hyperelastic incompressible materials. The problem is analysed within the framework of the finite strain theory of plane stress. Material induced nonlinearities are included through the use of the generalized neo-Hookean model which is characterized by three parameters which determine the small strain, "yielding" and "hardening" responses of the component(s). The structure of the near-tip stress and deformation fields is described and compared to a full-field finite element investigation. The consequences of the local results on the propagation behavior of a crack under general in-plane loading are outlined in the special case of a homogeneous sheet. The analytical results are corroborated by experimental observations obtained on natural rubber sheets.

1. INTRODUCTION

The complex nature of the singularity characterizing the linear asymptotic stress and strain fields (Williams, 1959) for a bimaterial interface crack and the associated "undesirable" phenomena (such as oscillatory stress fields, loss of consistency and crack interpenetration near the tip, etc.) have, for a long time, constituted a major obstacle to the analysis of the mechanics of interfacial failures. However, motivated by the increasing number of engineering applications involving bi-material and multi-material systems, numerous investigations have been dedicated to the interface fracture problem during the past decade.

This recent "surge of interest" has generated various approaches to cope with the aforementioned difficulties. The first solution (Comninou, 1977; Aravas and Sharma, 1991) consists of introducing a frictionless contact zone in the vicinity of the crack tip in order to resolve the inconsistency associated with the crack face overlapping. In another approach (Delale and Erdogan, 1988), the interface is given a finite thickness allowing for a smoother transition in the mechanical properties between those of the two (homogeneous) components. But the most commonly used method is associated with the concept of "small scale contact" (Rice, 1988) by which the various inconsistencies relative to the oscillatory solution are essentially ignored. This approach, which is motivated by the very small size of the "contact or overlapping zone" in most practical cases, has generated many linearly elastic investigations of various bimaterial situations [see, for example, the review by Hutchinson and Suo (1991)].

These studies have demonstrated some specific features of the bimaterial interface solution; in addition to the contact and overlapping problem already mentioned, the near-tip fields are characterized by a mode-mixity, the amplitude of which varies radially as the crack tip is approached. The lack of self-similarity of the local stress and deformation fields has been shown to have important consequences, for example on the analysis of the phenomena associated with the kinking of a crack off an interface (He and Hutchinson, 1989; Wang *et al.*, 1992; Geubelle and Knauss, 1991). Unlike in the homogeneous case, no fracture criterion can provide a unique prediction of the propagation angle and an additional length parameter corresponding to the length of the crack extension has to be introduced to resolve the uniqueness issue. Another feature of the bimaterial problem is the absence of a separable HRR-type solution within the framework of small strain deformation theory of plasticity (Shih, 1991) unless frictionless contact is assumed between the crack faces (Aravas and Sharma, 1991), thereby complicating the analysis of the effects of material related nonlinearities on the crack tip fields.

The objective of the present paper is to summarize a recent analysis of various nonlinear effects on the structure of the near-tip fields and on the propagation behavior of interface cracks. The investigation is motivated by previous work by Knowles and Sternberg (1983), later confirmed in a more general situation by Herrmann (1989), showing that the difficulties inherent in the small strain elastic and elasto-plastic analyses of the interface crack problem disappear when the kinematic assumption of infinitesimal deformations is relinquished; a separable asymptotic solution with real singularities is shown to exist, together with a smooth opening of the crack faces. The problem studied here concerns that of an interface crack between two sheets of hyperelastic materials. The present analysis extends the results of Knowles and Sternberg (1983) by adding material induced nonlinearities through the use of a more general model (the generalized neo-Hookean model) which captures some characteristic mechanical responses such as "yielding" and "hardening" behaviors.†

The second section contains some asymptotic results relative to the structure of the local fields, confirming the existence of a contact-free solution for an arbitrary choice of material parameters on both sides of the interface. Furthermore, it allows quantification of the nonlinear effects on the deformation fields by defining a relatively simple nonlinear mismatch parameter. The effects of the material parameters on the blunting of the crack are also outlined. The asymptotic results are compared with those of a full-field finite element analysis.

In addition to its "academic" interest when dealing with the various inconsistencies associated with the small strain asymptotics of bimaterial problems, the analysis presented here provides important results relative to the various fracture problems involving the presence of a non-negligible finite deformation zone near the crack tip. In the third section, the consequences of the structure of the near-tip fields on the propagation behavior of the crack is investigated analytically and numerically in the special case of homogeneous specimens. It is shown that, when a large deformation zone is allowed to develop near the tip of the crack prior to its propagation, the behavior under general in-plane mixed-mode conditions is expected to be quite different from the "brittle" (small strain) case. This result is corroborated by some experimental observations on sheets of natural rubber.

The paper focuses on the main results of the nonlinear analysis of the homogeneous and bimaterial fracture problem. Due to space constraint, most of the details relative to the analytical developments and the numerical computations have been left out. They can however be found in related papers (Geubelle and Knauss, 1992a,b,c, 1993).

2. STRUCTURE OF THE NEAR-TIP FIELDS FOR AN INTERFACE CRACK

This section summarizes some of the main results of an asymptotic analysis relative to the structure of the near-tip stress and deformation fields for a bimaterial interface crack. Figure 1 schematically represents the geometry of the generic fracture problem addressed here; a semi-infinite mathematically sharp crack is located along a straight interface between two sheets of isotropic, hyperelastic, incompressible materials. The standard axis conventions are adopted, as shown in Fig. 1. The analysis is carried out within the nonlinear finite deformation theory of plane stress, the main relations of which have been described by Knowles and Sternberg (1983).

The objective of the analysis outlined in the present section is to obtain an approximate expression, as the crack tip is approached, of the deformed position vector field $\mathbf{y}$ defined by the two-dimensional mapping of the sheet mid-plane Π:

$$\mathbf{y} = \hat{\mathbf{y}}(\mathbf{x}) = \mathbf{x}+\mathbf{u}(\mathbf{x}) \quad \text{on } \Pi \tag{1}$$‡

in which $\mathbf{u}$ is the displacement vector and $\mathbf{x}$ is the undeformed position vector. Associated

† The terms "yielding" and "hardening" are introduced here in correlation with the corresponding elastoplastic concepts (see Section 2) although the present analysis deals exclusively with hyperelastic components.

‡ The notations used throughout this paper are similar to those in Geubelle and Knauss (1992a,b,c).

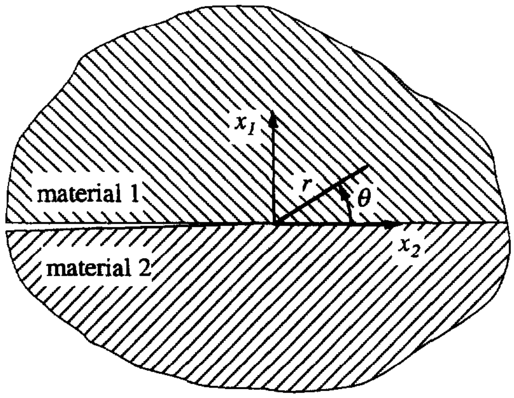

Fig. 1. Geometry of the interface fracture problem.

with the deformation are various kinematic quantities such as the deformation-gradient field $\mathbf{F} = \nabla \mathbf{y}$ and the scalar invariants $I = \mathrm{tr}(\mathbf{F}\mathbf{F}^{\mathrm{T}})$ and $J = \det \mathbf{F}$. Due to the incompressibility of the material, the in-plane Jacobian J is equal to the inverse of the out-of-plane (or transverse) stretch λ. The stress field can be equivalently expressed in terms of the Cauchy (or true) stress tensor $\boldsymbol{\tau}$ or the Piola (or nominal) stress tensor $\boldsymbol{\sigma}$ related to the former by

$$\boldsymbol{\sigma} = \boldsymbol{\tau}\mathbf{F}^{-\mathrm{T}} \quad \text{on } \Pi. \tag{2}$$

The equilibrium equations in absence of body forces are written as

$$\mathrm{div}\,\boldsymbol{\tau} = \mathbf{0}, \quad \boldsymbol{\tau} = \boldsymbol{\tau}^{\mathrm{T}} \quad \text{on } \Pi^* = \hat{\mathbf{y}}(\Pi). \tag{3}$$

The materials on both sides of the interface are assumed to behave according to the generalized neo-Hookean (GNH) model introduced by Knowles (1977) to study the effect of both geometrical and material nonlinearities on the structure of the near-tip fields in the homogeneous anti-plane shear case. This model is characterized by three material constants μ, b and n entering the expression of the plane stress elastic potential

$$U(I, J) = \frac{\mu}{2b}\left\{\left[1+\frac{b}{n}(I+J^{-2}-3)\right]^n - 1\right\} \tag{4}$$

where I and J have been introduced above. As shown in Figs 2(a) and 2(b) which illustrate the uniaxial stress–strain behavior of this class of materials, each constant corresponds to a basic characteristic of the material response; the shear modulus μ governs the linear elastic regime, the "yielding" parameter b determines the amount of linearity while n corresponds to the "hardening" behavior. Throughout this analysis, only values of n greater than $1/2$ will be considered, in order to guarantee the ellipticity of the equilibrium equations. As shown in Fig. 2(a), the mechanical response of the material reaches an asymptote in the limiting case ($n = 1/2$), henceforth referred to as the "perfectly plastic" situation. The relations between the stress components and the deformed coordinates are

$$\begin{aligned} \sigma_{\alpha\beta} &= \mu A^{n-1}\{y_{\alpha,\beta} - \lambda^3\,\varepsilon_{\alpha\mu}\varepsilon_{\beta\nu}y_{\mu,\nu}\} \\ \tau_{\alpha\beta} &= \mu A^{n-1}\,\{y_{\alpha,\gamma}y_{\beta,\gamma} - \lambda^2\delta_{\alpha\beta}\} \end{aligned} \tag{5}$$

where $A = 1 + b\,(I+\lambda^2-3)/n$ and $\varepsilon_{\alpha\beta}$ and $\delta_{\alpha\beta}$ are the two-dimensional alternator and identity operator, respectively.

In the remainder of the section, we outline the main steps leading to a near-tip solution of the fracture problem illustrated in Fig. 1, which is consistent with the various relations mentioned above, in the particular case where the two sheets have the same "hardening"

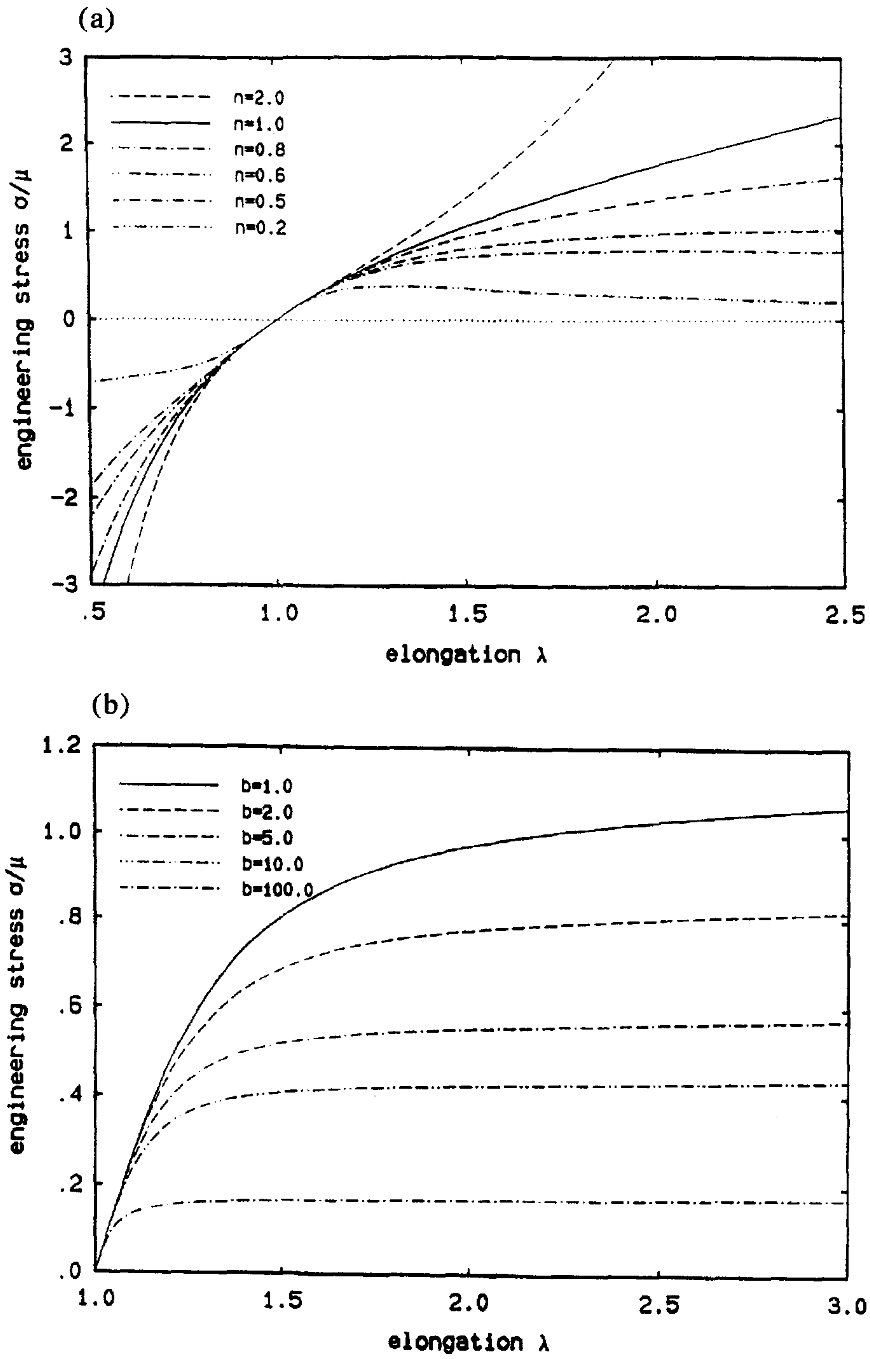

Fig. 2. Uniaxial response of the GNH materials: (a) effect of the "hardening" exponent n; (b) effect of the "yielding" parameter b.

behavior (i.e. $n^{(1)} = n^{(2)} = n$).† Then the results of the general situation $n^{(1)} \neq n^{(2)}$ will be summarized and commented on.

2.1. *Interface crack between two GNH sheets of same hardening characteristics*

We start the local analysis by assuming a separable form of the near-tip deformation field

† The superscript in parenthesis (k) is the material index, takes the values 1 (for the top sheet) and 2 (for the bottom sheet) and is *never* summed over. It is often omitted for brevity purposes unless required for clarity.

$$y_\alpha^{(k)}(r,\theta) \sim r^{m^{(k)}} v_\alpha^{(k)}(\theta) \tag{6}$$

where the polar coordinates r and θ have been defined in Fig. 1 and where the exponents $m^{(k)}$ are required to be real in order to avoid the oscillatory singularity arising in the linearized local solution. The approximation (6) has to satisfy zero-traction boundary conditions along the crack faces

$$\sigma_{\alpha 2}(r, \pm\pi) = 0 \tag{7}$$

and the displacement and traction continuity conditions along the interface

$$\begin{aligned} y_\alpha^{(1)}(r,0^+) &= y_\alpha^{(2)}(r,0^-) \\ \sigma_{\alpha 2}^{(1)}(r,0^+) &= \sigma_{\alpha 2}^{(2)}(r,0^-). \end{aligned} \tag{8}$$

The latter conditions can be shown to lead to

$$m^{(1)} = m^{(2)} = m. \tag{9}$$

The value of the exponent m can be determined by investigating the leading order of the integrant entering the definition of the conservation integral.

$$\mathscr{J} = \int_\Gamma (Un_1 - \sigma_{\alpha\beta} n_\beta y_{\alpha,1})\,\mathrm{d}s \tag{10}$$

where Γ is any regular contour of outward normal $\mathbf{n}$ surrounding the crack tip and crossing the interface. Substituting the assumed form of the asymptotic field (6) into (10) yields, with the aid of (4) and (5),

$$m = 1 - 1/2n. \tag{11}$$

Note that, for $n > 1/2$, m satisfies the inequalities $0 < m < 1$, which guarantee finite displacements but singular strains at the crack tip. The angular functions $v_\alpha^{(k)}(\theta)$ introduced in (6) are obtained by solving the asymptotic form of the equilibrium equations

$$(n-1)\frac{\partial I}{\partial x_\beta} y_{\alpha,\beta} + I\nabla^2 y_\alpha = 0 \tag{12}$$

which yield

$$v_\alpha^{(k)}(\theta) = a_\alpha^{(k)} f(\theta; n) \tag{13}$$

where $a_\alpha^{(k)}$ are four undetermined constants and $f(\theta;n)$ is represented† in Fig. 3 for various values of the "hardening" parameter n. The solution (13) satisfies the zero-traction conditions along $\theta = \pm\pi$ and the displacement continuity along the interface. The last boundary condition yields an additional relation between the constants $a_\alpha^{(1)}$ and $a_\alpha^{(2)}$, such that the final form of the first term of the near-tip approximation is

$$y_\alpha(r,\theta) \sim a_\alpha j(\theta) f(\theta;n)\, r^m \tag{14}$$

where a_α are two constants left undetermined by the local analysis and $j(\theta)$ is the angular step function

† A closed form of $f(\theta, n)$, first derived by Knowles and Sternberg (1973), can be found in eqn (3.19) of Geubelle and Knauss (1992a).

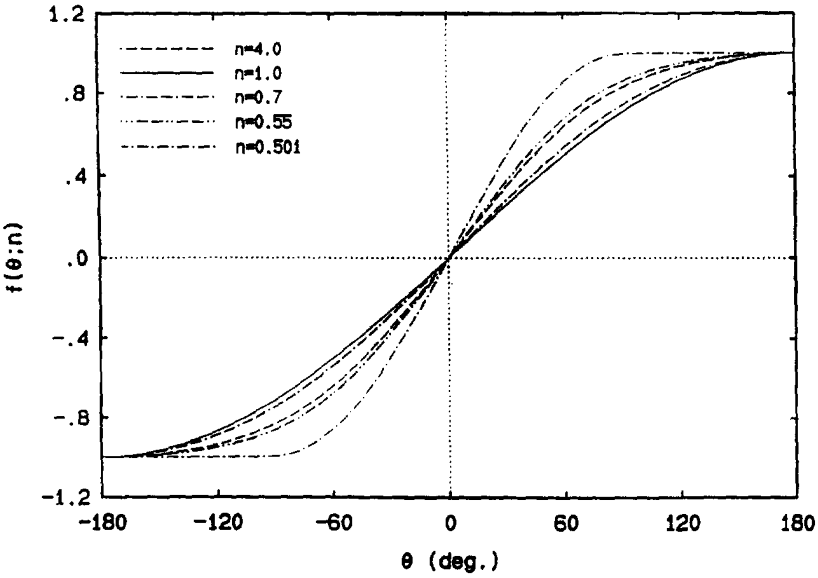

Fig. 3. $f(\theta; n)$ versus θ for various values of n.

$$j(\theta) = \begin{cases} 1 & \text{for } 0 \leqslant \theta \leqslant \pi \\ \xi & \text{for } -\pi \leqslant \theta < 0 \end{cases} \tag{15}$$

in which ξ is the nonlinear mismatch parameter, function of the five material constants,

$$\xi = \left[\frac{\mu^{(1)}}{\mu^{(2)}}\left(\frac{b^{(1)}}{b^{(2)}}\right)^{n-1}\right]^{1/(2n-1)}. \tag{16}$$

The nonlinear mismatch parameter provides a very concise way to quantify the effects of material induced nonlinearities on the strain distribution across the interface. While the neo-Hookean case ($n = 1$) suggests a deformation distribution inversely proportional to the ratio of material stiffnesses, as is the case in the linearized solution, (16) indicates that, as n decreases, the strains will be more unevenly distributed across the interface and, in the limiting case of the "perfectly plastic" situation ($n \to 1/2$), all the deformations are concentrated in the weaker component ($\xi \to 0$ or $\xi \to \infty$).

The asymptotic solution obtained so far is however not consistent; the value of the in-plane Jacobian J is identically zero everywhere, leaving the out-of-plane stretch λ undetermined. An additional term is therefore needed; it is assumed to have a similar separable form

$$y_\alpha(r, \theta) \sim a_\alpha j(\theta) f(\theta; n) r^m + r^{s^{(k)}} w_\alpha^{(k)}(\theta) \tag{17}$$

where we require the exponents $s^{(k)}$ to be real and greater than m. As was the case before, matching conditions along $\theta = 0$ yield

$$s^{(1)} = s^{(2)} = s. \tag{18}$$

The value of s and the expression for the angular functions $w_\alpha^{(k)}(\theta)$ are found by solving the boundary value problem arising from substituting (17) into the asymptotic form (12) of the equilibrium equations, the traction-free conditions along the crack faces and the continuity requirements along the interface. Various higher-order terms can be determined in the process, yielding

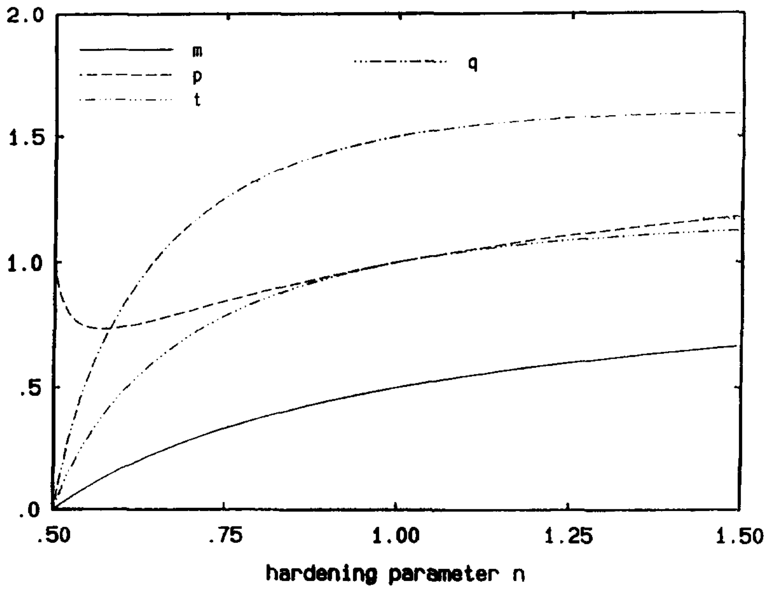

Fig. 4. Asymptotic exponents m, p, t and q versus the "hardening" parameter n.

$$y_\alpha(r,\theta) \sim a_\alpha r^m j(\theta) f(\theta;n) + c\varepsilon_{\alpha\beta} a_\beta r^p g(\theta;n) + k a_\alpha r^t l(\theta;n) + d a_\alpha r^q j(\theta)\, h(\theta;n) \tag{19}$$

where c, k and d are undetermined constants; $\varepsilon_{\alpha\beta}$ is the two-dimensional alternator and the exponents p, t and q are functions of the "hardening" parameter n (Fig. 4). The Jacobian of the in-plane transformation can be shown to behave asymptotically as

$$J = \lambda^{-1} \sim r^{m+p-2} j(\theta)(p\dot{f}g - mf\dot{g}) \tag{20}$$

and is illustrated in Fig. 5 for the mismatch parameter $\xi = 3$. The local solution is thus characterized by an extreme thinning of the sheet as the crack tip is approached.

A major result of the present local analysis is the confirmation of the existence of an oscillation-free, contact-free solution, as had been first shown by Knowles and Sternberg (1983) in the particular case of a neo-Hookean bimaterial sheet ($n = 1$). The crack faces

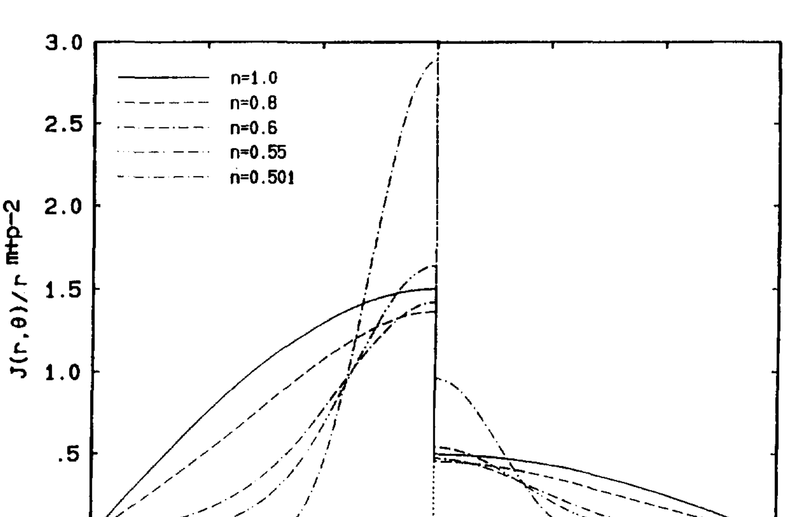

Fig. 5. Angular variation of the in-plane Jacobian for $\zeta = 3$ for various values of n.

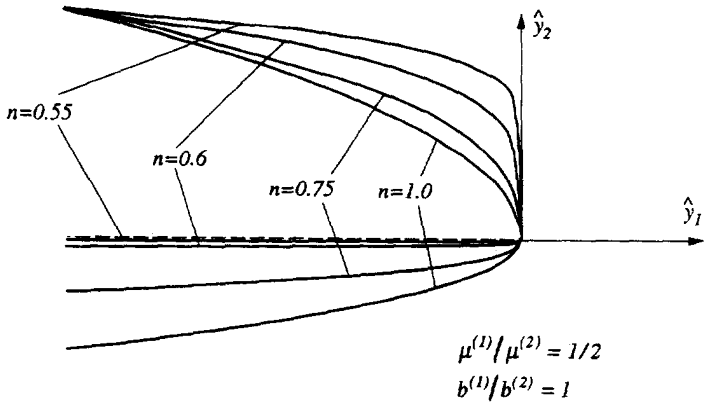

Fig. 6. Schematic representation of the effect of n on the shape of the deformed crack for 4 values of n and for $\mu^{(2)} = 2\mu^{(1)}$, $b^{(1)} = b^{(2)}$.

are found to open smoothly for all loading conditions. Furthermore, relation (20), together with the description of the shape of the deformed crack

$$y_1(r, \pm\pi) = c^{\text{ste}} \left(\frac{y_2(r, \pm\pi)}{j(\pm\pi)} \right)^{p/m}, \tag{21}$$

which is schematically illustrated in Fig. 6, allow us to get a better understanding of the effect of n on the crack blunting. As seen in Fig. 5, most of the deformations get concentrated in the first and fourth quadrants ($-\pi/2 \leqslant \theta \leqslant \pi/2$) and the deformed crack adopts a square-like shape as the hardening characteristics of the material get weaker. Figure 6 also illustrates the aforementioned phenomenon of strain concentration in the weaker component, as quantified by the nonlinear mismatch parameter ξ. Further discussion of the local approximation will be given in the next paragraph which summarizes the results of the general bimaterial situation, allowing for an arbitrary choice of the hardening exponents ($n^{(1)} \neq n^{(2)}$).

2.2. *Near-tip fields in the general bimaterial situation*

Although it is somewhat more complex, the solution to the general problem† is very similar to the reasoning described in the previous section. The final form of the near-tip deformation fields can be written as

$$\mathbf{y} = \mathbf{Q}\hat{\mathbf{y}} \tag{22}$$

where $\mathbf{Q}$ is a two-dimensional rotation matrix

$$[Q_{\alpha\beta}] = \frac{1}{1 + a_{12}^2} \begin{pmatrix} 1 & a_{12} \\ -a_{12} & 1 \end{pmatrix} \tag{23}$$

and $\hat{\mathbf{y}}$ is the "reference" (or "canonical") field

$$\begin{cases} \hat{y}_1^{(1)}(r,\theta) \sim c^{(1)} a^{(1)} r^p g^{(1)}(\theta) \\ \hat{y}_2^{(1)}(r,\theta) \sim a^{(1)} r^{m^{(1)}} f^{(1)}(\theta) + a^{(1)} r^{2m^{(1)} - m^{(2)}} \bar{w}^{(1)}(\theta) + k^{(1)} a^{(1)} r^t l^{(1)}(\theta) \end{cases} \tag{24}$$

$$\begin{cases} \hat{y}_1^{(2)}(r,\theta) \sim c^{(2)} a^{(2)} r^p g^{(2)}(\theta) \\ \hat{y}_2^{(2)}(r,\theta) \sim a^{(2)} r^{m^{(2)}} f^{(2)}(\theta) + a^{(1)} r^{m^{(1)}} \bar{w}^{(2)}(\theta) + k^{(2)} a^{(2)} r^t l^{(2)}(\theta) \end{cases} \tag{25}$$

† Without loss of generality, we will assume that $n^{(1)} \geqslant n^{(2)}$.

in which a_{12} is a "mode-mixity parameter" associated with the first asymptotic term; c, a and k are constants; the exponents $m^{(k)}$, p and t are presented in Fig. 7 and depend on the hardening parameters $n^{(k)}$. Various relations can be derived between the undetermined constants $a^{(k)}$, $c^{(k)}$ and $k^{(k)}$ through the continuity conditions along the interface [see Geubelle and Knauss (1992c) for more details].

Many characteristics of the solution described by (22)–(25) are similar to those obtained in the special case $n^{(1)} = n^{(2)}$ summarized above; the existence of an oscillation-free and contact-free solution is confirmed and the effects of the "hardening" parameters on the crack tip blunting and the strain distribution are also captured. The general situation presents however two major differences with the simpler case; the mismatch in hardening exponents across the interface generates new terms in the asymptotic expansion of order $O(r^{2m^{(2)}-m^{(1)}})$ for the top sheet and $O(r^{m^{(1)}})$ for the bottom one and the intensity of the loading quantified by the value of the $\mathscr{J}$-integral (10) can be shown to affect the ratio of the first term amplitudes $a_\alpha^{(k)}$ as

$$a_\alpha^{(1)} = \xi^{(1-2n^{(2)})/(2n^{(1)}-1)} \left(\frac{\mathscr{J}}{\mu^{(2)}(b^{(2)})^{n^{(2)}-1}\bar{\mathscr{J}}(n^{(2)}, m^{(2)})} \right)^{(n^{(2)}-n^{(1)})/(n^{(2)}(2n^{(1)}-1))} a_\alpha^{(2)} \quad (26)$$

where ξ is a function of the six material parameters $(\mu^{(k)}, b^{(k)}, n^{(k)})$ which reduces to the expression (16) of the nonlinear mismatch parameter in the case $n^{(1)} = n^{(2)}$. The particular case of a GNH sheet bonded to a rigid substrate has been discussed in detail in Geubelle and Knauss (1992b).

As indicated by (22), the near-tip deformation field $\hat{\mathbf{y}}$, the rotation depending on the far-field loading and the material and geometrical characteristics of the bimaterial sheet. This fact has an important consequence on the propagation behavior of the crack and will

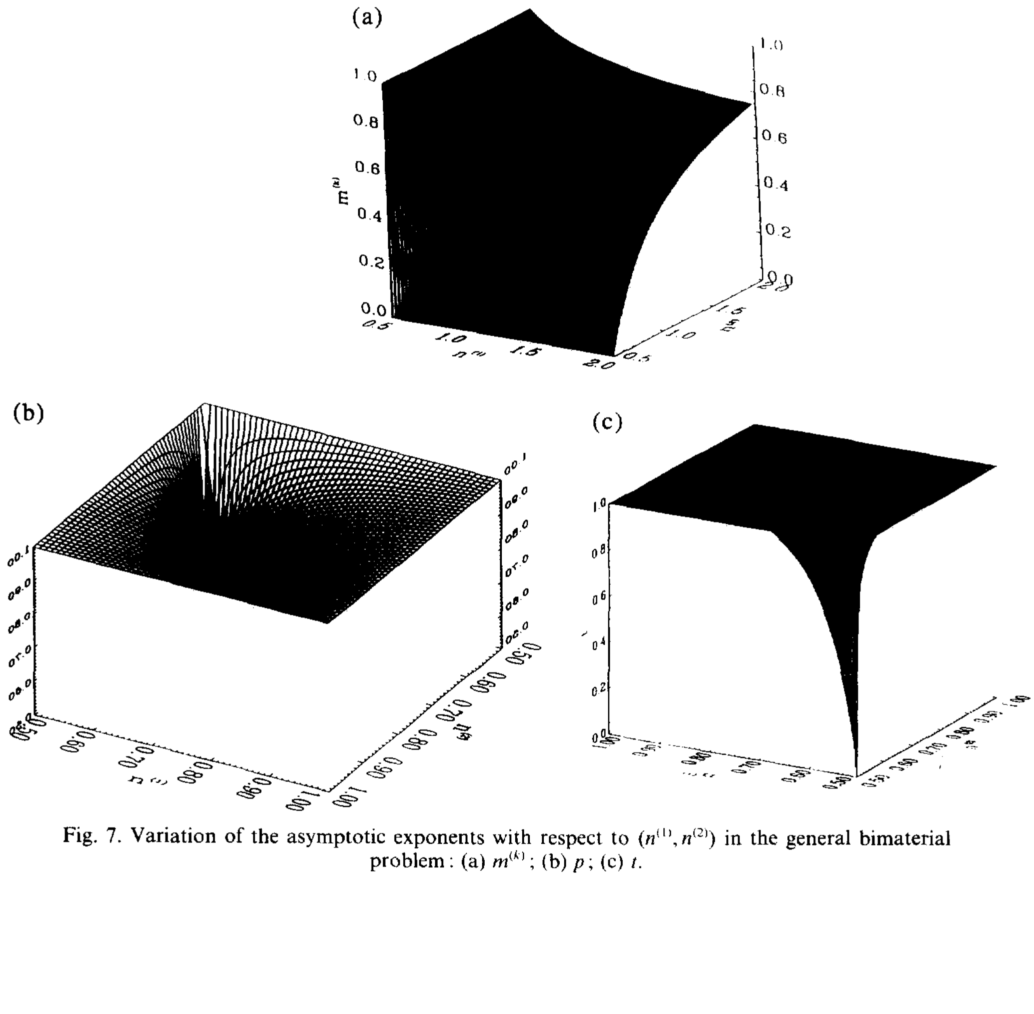

Fig. 7. Variation of the asymptotic exponents with respect to $(n^{(1)}, n^{(2)})$ in the general bimaterial problem: (a) $m^{(k)}$; (b) p; (c) t.

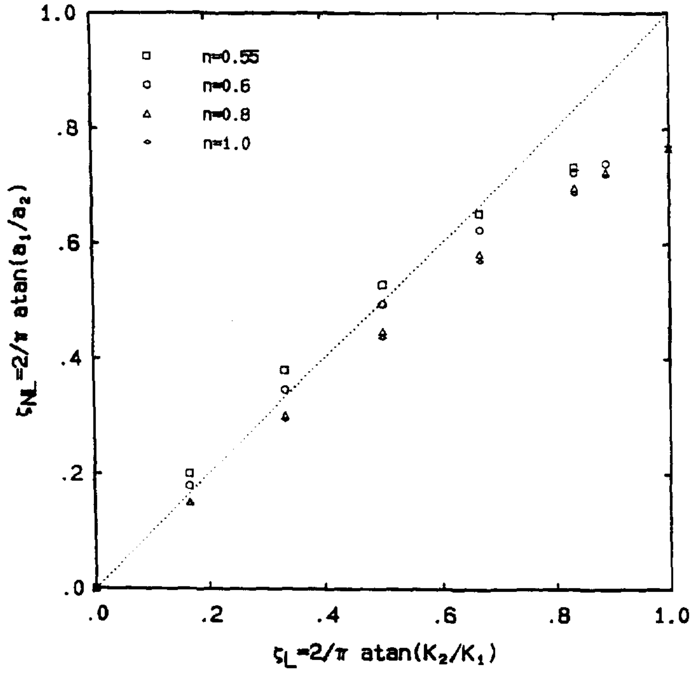

Fig. 8. Relation between the linear (ζ_L) and nonlinear (ζ_{NL}) mode-mixity parameters for various values of n in the homogeneous case.

be examined in greater detail in the next section in the particular case of a homogeneous sheet of GNH material.

3. NONLINEAR INVESTIGATION OF CRACK PROPAGATION

In the homogeneous situation, the near-tip field (22)–(25) reduces to

$$\mathbf{y} = \mathbf{Q}\hat{\mathbf{y}} \tag{27}$$

where the two-dimensional rotation matrix $\mathbf{Q}$ has a form similar to (23) and the reference deformation field $\hat{\mathbf{y}}$ is the symmetric (mode I) field

$$\begin{cases} \hat{y}_1 \sim cr^p g(\theta;n) \\ \hat{y}_2 \sim ar^m f(\theta;n) + kr^t l(\theta;n) + dr^q h(\theta;n) \end{cases} \tag{28}$$

in which the exponents m, p, t and q have been shown in Fig. 4 and a, c, k and d are constants left undetermined by the local analysis. The scalar a can however be related to the far-field loading through the value of the $\mathcal{J}$-integral as

$$\mathcal{J} = 2\mu b^{n-1} a^{2n} m^{2n-1} n^{1-n} \pi/4. \tag{29}$$

The relation between the nonlinear mode-mixity, defined as $\zeta_{NL} = 2\tan^{-1}(a_{12})/\pi$ and the far-field loading can only be obtained numerically. Following a concept similar to that of "small scale yielding" [adopted by Shih (1974) in his study of the mixed-mode HRR fields in the homogeneous case], a boundary layer approach† can be used to determine the relationship between ζ_{NL} and its linear counterpart $\zeta_L = 2\tan^{-1}(K_2/K_1)/\pi$ where K_α are the (linear) stress intensity factors, as illustrated in Fig. 8 for various values of n. This result, together with (27) and (28), confirms the fact, first recognized by Stephenson (1982) and

† This method consists of numerically solving a full-field nonlinear fracture problem on a circular domain along the boundary of which conditions corresponding to a (linear) K-field are applied.

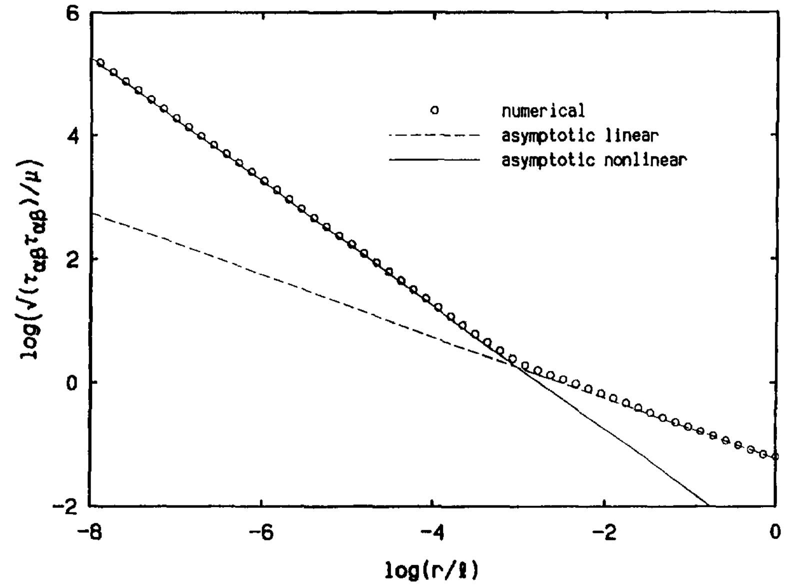

Fig. 9. Radial variation of the norm of the Cauchy stress tensor ahead of the crack tip and definition of the size r_{NL} of the nonlinear zone.

further analysed by Knowles (1981), that a purely antisymmetric (mode II) solution cannot arise in the framework of finite deformation elastic theory; the crack is due to always open in the vicinity of its tip.

A comparison of the nonlinear and linear asymptotic solutions also allows definition of a length scale characteristic of the size of the nonlinear zone. As shown in Fig. 9, the radial variation of the Cauchy stress tensor norm ahead of the crack shows a sharp transition between the $r^{-1/2}$ singularity of the small strain solution and the stronger r^{-1} singularity associated with the nonlinear analysis. The length parameter r_{NL} defined by the intersection of the two asymptotics can be shown to be related to $\mathcal{J}$ through

$$\frac{r_{NL}}{l} = \frac{1}{3\pi n^2}\frac{\mathcal{J}}{\mu l}. \tag{30}$$

As mentioned above, the "rotational property" (27) of the near-tip field has a fundamental impact on the propagation behavior of the crack; it seems to indicate that the crack will always tend to propagate in the direction of its original plane regardless of the far-field loading conditions.† The problem was studied numerically through the finite element method and the boundary layer approach. The propagation criterion used here is the maximum energy release rate criterion which suggests that the path [characterized by the extension length Δl and the kink angle ω measured with respect to the undeformed crack plane (Fig. 10)] adopted by the crack under general in-plane loading conditions [characterized by the mode-mixity $\gamma = \tan^{-1}(K_2/K_1)$ of the applied "far-field" K-field] is the path that maximizes the energy release rate $G(\omega, \Delta l)$ defined as the variation of the total potential energy of the fracture specimen for an infinitesimally small crack extension ($\Delta l \to 0$). As was the case in the asymptotic analysis, the material model used in the numerical investigation is the generalized neo-Hookean one. Figure 10 illustrates the results of the numerics by presenting the ratio of the "nonlinear" energetically most favorable kink angle ω^* normalized by its "linear" counterpart ω_L^* (Palaniswamy and Knauss, 1978) as a function of the crack extension length Δl [normalized by the size of the finite deformation zone r_{NL} defined in (30)] for various values of the far-field mode-mixity γ. As predicted by the asymptotics, Fig. 10 shows that, except in the case ($\gamma = 0$) (mode I loading) for which

† It has to be noted that this property has been shown by Le (1992) to also apply to the plane strain analysis for a completely different class of (compressible) hyperelastic materials.

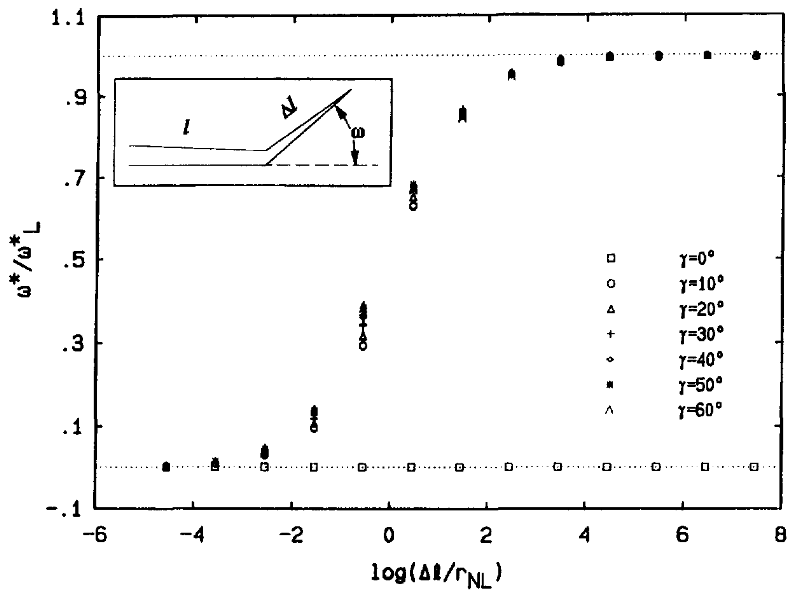

Fig. 10. Nonlinear kink angle ω^* (normalized by the corresponding linear value ω_L^*) versus the crack extension length $\Delta l/r_{NL}$ for various values of the far-field mode-mixity γ.

the predicted kink angle is always zero by symmetry, a sharp transition is observed between the γ-dependent linear value ω_L^* and the γ-independent nonlinear value $\omega^* = 0$.

Most experimental investigations of crack propagation under mixed-mode in-plane conditions available to date involve brittle situations in which the finite strain effects are quasi negligible and seem therefore to conform with the small strain results (Palaniswamy and Knauss, 1978). A recent series of tests (Hodowany-Stone and Montilla, 1993) performed on natural rubber sheets subjected to mixed-mode loading conditions showed a behavior very different to that observed in previously recorded experiments. As schematically illustrated in Fig. 11(a), instead of showing the steep kink angle observed in brittle specimens, the crack extended first in the direction of its original line ($\omega = 0$) then curved progressively away toward a direction perpendicular to the loading axis. This behavior was observed for all inclinations of the initial crack with respect to the applied loading axis (i.e. for all applied mode-mixities), thereby corroborating the asymptotic and numerical results. A typical fractured specimen is shown in Fig. 11(b). Details on the bimaterial situation can be found in Geubelle and Knauss (1993) where it is shown that, unlike in the linearized case, a unique prediction of the crack propagation angle is obtained, which depends on the material combination.

4. CONCLUSIONS

This paper has described some results of an asymptotic and numerical investigation of the effects of geometry and material induced nonlinearities on the structure of the near-tip fields in various homogeneous and bimaterial situations for the particular class of generalized neo-Hookean materials. The main results can be summarized as follows:

(i) The local fields are characterized by multiple real singularities, even in the general bimaterial situation, which indicates the existence of an oscillation-free and contact-free solution with a smooth opening of the crack faces.

(ii) The singularities are stronger than in the linearized analysis, especially as the material(s) present(s) a weak "hardening" behavior.

(iii) The asymptotics of the interface crack problem is characterized by a nonlinear mismatch parameter which quantifies the distribution of the strains on both sides of the interface.

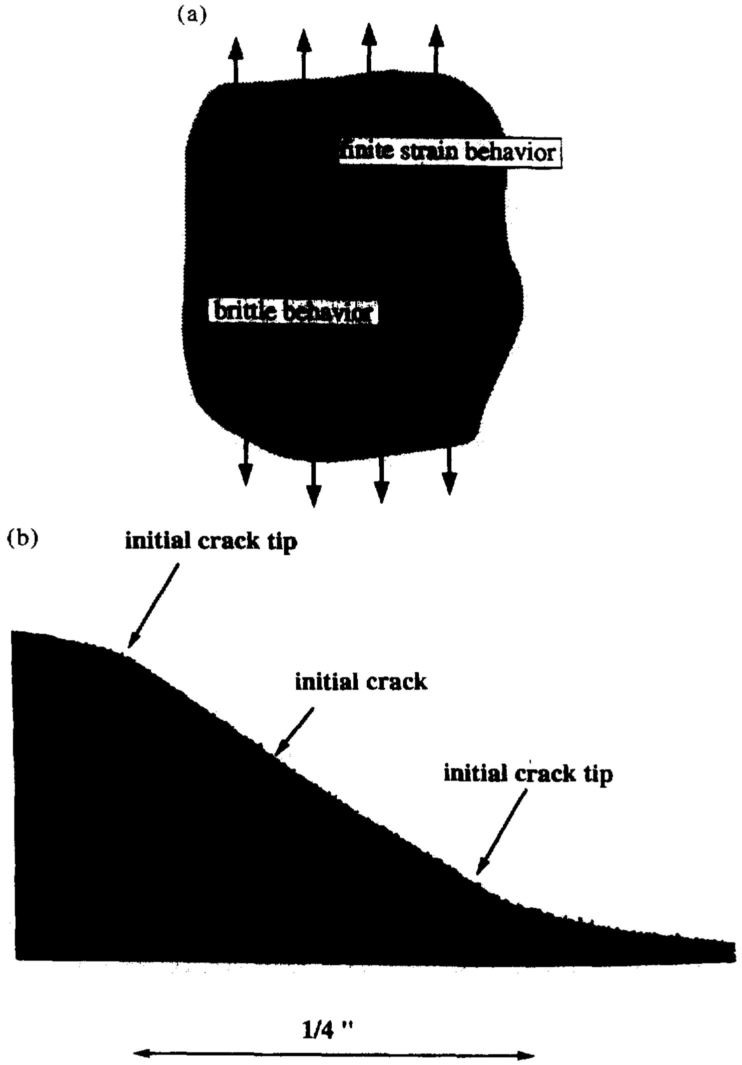

Fig. 11. Experimental observations on natural rubber sheets: (a) schematic illustration of typical small strain (brittle) and large strain behaviors; (b) view of the fracture specimen after crack propagation.

(iv) The near-tip deformation field is shown to correspond to the rotation of a simpler "reference field", the rotation depending on the far-field loading conditions and the material properties of the component(s). A boundary layer numerical analysis of the homogeneous case has yielded a relation between the "far-field" mode-mixity and the rotation parameters, confirming the impossibility of a pure mode II solution within the nonlinear elastic theory.

(v) Finally, the crack propagation problem has been investigated, showing that a very different behavior is to be expected when a large deformation zone is allowed to develop prior to the propagation. The results obtained analytically and numerically have been corroborated by experimental observations of mixed-mode crack propagation in sheets of natural rubber.

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